TEMPERATURE PATTERNS IN SOILS WITH VARIABLE THERMOPHYSICAL COEFFICIENTS WITH GIVEN TEMPERATURE CONDITIONS IN GREENHOUSES

L. E. Rybakova, B. M. Makhmaerov,
B. Kh. Khairuddinov, and P. V. Tsoi

UDC 536.2.01

An analytic method is proposed for solving nonstationary heat conduction with transport coefficients dependent on the coordinates. The temperature distributions in soils have been examined for harmonic and exponential laws of variation for the air temperature in greenhouses.

A major factor determining the growth and development of plants in a greenhouse is the provision of the best thermal conditions in the root layer of the soil. The thermal conditions in the soils of solar greenhouses are produced mainly by solar radiation and the warmth of the air within the greenhouse [1]. It is very complicated to find the temperature distribution in the soil, where various transport factors are involved (convection, conduction, and radiation) [2].

The use of an equivalent thermal conductivity [2] is an effective means of formulating a model for heat transfer in the soil that enables one to avoid some of the difficulties in solving the transport equations. The approach enables one to perform a theoretical study of the temperature distribution in a soil by the solution of a single heat-conduction equation with variable thermophysical coefficients dependent on the coordinates and time.

The researches of [2] have yielded some empirical relationships for the thermophysical characteristics of soils. Therefore, the theoretical aspect of the problem is that one has to obtain the corresponding solutions to the equivalent equation of thermal conduction for these relationships and thus provide a method for calculating the temperature distribution in particular soils.

Here we present a simple and reliable analytic method of handling problems in nonstationary thermal conduction with variable and constant transport coefficients, which was fairly fully developed in [3]. The use of this method in the analytical theory of soils enables one to solve new problems in a simple fashion.

Measurements show that the maximum penetration depth of the temperature perturbation in a soil in response to diurnal variations in air temperature in a greenhouse may be taken as $l = 0.5$ m. Let the air temperature follow the law $\varphi(t)$, while the heat transfer between the air and the surface of the soil follows Newton's law:

$$\left(\lambda \frac{\partial T}{\partial x}\right)_{x=0} = \alpha [T(x, t) - \varphi(t)]_{x=0}. \tag{1}$$

Here α is the effective heat-transfer coefficient, which includes the fraction of the heat flux obtained by the soil from solar radiation. Then the temperature distribution $T(x, t)$ within the soil $(0 \le x \le l)$ is found by solving

$$c(x, m_1) \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left[\lambda(x, m_2) \frac{\partial T}{\partial x}\right], \ T(x, 0) = T_0, \tag{2}$$

$$\left(\lambda_0 \frac{\partial T}{\partial x}\right)_{x=0} = \alpha [T(x, t) - \varphi(t)]_{x=0}, \ \left(\lambda_l \frac{\partial T}{\partial x}\right)_{x=l} = 0. \tag{3}$$

The effective variable thermal conductivity and specific heat are often approximated in the following form [2]:

$$\lambda(x, m_2) = \lambda_0 [1 + f_\lambda(x, m_2)], \ c(x, m_1) = c_0 [1 + f_c(x, m_1)],$$

Tadzhik Polytechnical Institute, Dushanbe. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 42, No. 2, pp. 313-320, February, 1982. Original article submitted May 25, 1981.

 0022-0841/82/4202-0220$07.50

where the functions f_λ and f_c satisfy the conditions

$$\lim_{x\to 0} f_\lambda(x, m_2) = 0, \ \lim_{x\to 0} f_c(x, m_1) = 0, \ \lim_{m_2\to 0} f_\lambda(x, m_2) = \text{const},$$
$$\lim_{m_1\to 0} f_c(x, m_1) = \text{const}.$$

We introduce the relative coordinate $\xi = x/l$ and the dimensionless time $\text{Fo} = a_0 t/l^2$, where $a_0 = \lambda(0, m_2)/c(0, m_1)$; then the problem of (2) and (3) reduces to

$$[1 + f_c(\xi, \beta_1)]\frac{\partial T}{\partial \text{Fo}} = \frac{\partial}{\partial \xi}\left[(1 + f_\lambda(\xi, \beta_2))\frac{\partial T}{\partial \xi}\right], \ T(\xi, 0) = T_0, \tag{4}$$

$$\left\{\frac{\partial T}{\partial \xi} - \text{Bi}\, T(\xi, \text{Fo})\right\}_{\xi=0} = -\text{Bi}\,\varphi(\text{Fo}), \ \left(\frac{\partial T}{\partial \xi}\right)_{\xi=1} = 0, \tag{5}$$

where $\text{Bi} = \alpha l/\lambda_0$; $\beta_1 = m_1/l$; $\beta_2 = m_2/l$; the parameters β_1 and β_2 are the correcting parameters for the nonuniformity in the thermophysical coefficients. As β_1 and β_2 become smaller, the changes in the thermal conductivity and specific heat with the coordinate x become weaker, and for $\beta_1 = \beta_2 = 0$ the problem of (4) and (5) becomes that of thermal conduction with constant transport coefficients.

We denote by $\bar{T}(\xi, s)$ the integral Laplace transform of the temperature distribution, i.e.,

$$\bar{T}(\xi, s) = \int_0^\infty T(\xi, \text{Fo}) \exp(-s\,\text{Fo})\, d\,\text{Fo}.$$

Then with the conversion formula $\partial T/\partial \text{Fo} \doteqdot s\bar{T}(\xi, s) - T(\xi, 0)$ the problem of (4) and (5) after Laplace transformation amounts to solving the boundary-value problem

$$\frac{d}{d\xi}\left[(1 + f_\lambda(\xi, \beta_2))\frac{d\bar{T}}{d\xi}\right] - [s\bar{T}(\xi, s) - T_0](1 + f_c(\xi, \beta_1)) = 0, \tag{6}$$

$$\left\{\frac{d\bar{T}}{d\xi} - \text{Bi}\, T(\xi, s)\right\}_{\xi=0} = -\text{Bi}\,\bar{\varphi}(s), \ \left(\frac{d\bar{T}}{d\xi}\right)_{\xi=1} = 0. \tag{7}$$

One obtains a very complicated functional dependence for the exact solution of this boundary-value problem even for simple particular cases, and it is not always possible to find a way of converting the transform $\bar{T}(\xi, s)$ to the original $T(\xi, \text{Fo})$. Therefore, an approximate method of solving (6) and (7) has been developed such as to provide a solution to the initial problem as a simple analytic formula, which is of considerable practical importance. Developments in this area include orthogonal projection (Galerkin's method) for (6) and (7), whose essence is as follows. An approximate solution is found as an element in functional space whose bases $\psi_1(\xi)$, $\psi_3(\xi)$, ..., $\psi_n(\xi)$ satisfy the homogeneous boundary conditions of (7), i.e.,

$$\left\{\frac{d\psi_k}{d\xi} - \text{Bi}\,\psi_k\right\}_{\xi=0} = 0, \ \left\{\frac{\partial\psi_k}{d\xi}\right\}_{\xi=1} = 0, \ k = 1, 2, \ldots, n,$$

in a linear-composition family of the form

$$\bar{T}_n(\xi, s) = \bar{\varphi}(s) + \sum_{k=1}^{n} \bar{a}_k(s)\,\psi_k(\xi). \tag{8}$$

We taken the following as basis coordinate functions:

$$\psi_1(\xi) = \frac{\text{Bi} + 2}{\text{Bi}} - (1 - \xi)^2, \ \psi_k = (1 - \xi)^2 \xi^{2(k-1)}, \ k \geqslant 2,$$

for which (8) satisfies the boundary conditions of (7) exactly.

The transform coefficients $\bar{a}_k(s)$ are projections of the vector $T_n(\xi, s) - \bar{\varphi}(s)$ on the coordinate axes of the functional space and are found from the condition of orthogonality for the discrepancies in (6) as found by substituting (8) for $\bar{T}(\xi, s)$ for all the basis functions $\psi_j(\xi)$:

$$\int_0^1 \left\{\frac{d}{d\xi}\left[(1 + f_\lambda(\xi, \beta_2))\frac{d\bar{T}_n}{d\xi}\right] - [s\bar{T}_n(\xi, s) - T_0](1 + f_c(\xi, \beta_1))\right\}\psi_j d\xi = 0,$$
$$j = 1, 2, \ldots, n.$$

After integration with respect to the variable ξ for particular forms of f_λ and f_c, the system becomes

$$\left\{\sum_{k=1}^{n} (A_{jk} + B_{jk}s)\,\bar{a}_k(s) = [T_0 - s\bar{\varphi}(s)]\,D_j\right\},\quad j = 1, 2, \ldots, n, \tag{9}$$

where

$$A_{jk} = A_{kj} = -\int_0^1 \frac{d}{d\xi}\left[(1+f_\lambda)\frac{d\psi_k}{d\xi}\right]\psi_j(\xi)\,d\xi > 0;$$

$$B_{jk} = B_{kj} = \int_0^1 (1+f_c)\,\psi_j\psi_k d\xi;\quad D_j = \int_0^1 (1+f_c)\,\psi_j d\xi.$$

We determine the coefficients $\bar{a}_k(s)$ from (9) from Cramer's formula:

$$\bar{a}_k(s) = \frac{\Delta_k(s)[T_0 - s\bar{\varphi}(s)]}{\Delta(s)}, \tag{10}$$

where (8) gives the best approximation to the solution to (6) and (7), and we transfer to the region of the originals to find the solution to the initial problem as

$$T_n(\xi,\ \mathrm{Fo}) = \varphi(\mathrm{Fo}) + \sum_{k=1}^{n} a_k(\mathrm{Fo})\,\psi_k(\xi), \tag{11}$$

where $\Delta(s) = |A + Bs|$ is the basic determinant of (9); $\Delta_k(s)$ is the determinant obtained by replacing column k by the coefficients $D_1, D_2, \ldots, D_n$ in the determinant $\Delta(s)$. The roots of $\Delta(s) = 0$ are always simple and negative. We denote them by $s_1, s_2, \ldots, s_n$ in order of increasing magnitude. Then the theory for transforming a fractionally rational function with simple poles and the convolution theorem [4] give us that

$$a_k(\mathrm{Fo}) = \sum_{k=1}^{n} \frac{\Delta_k(s_i)}{\Delta'(s_i)} \int_0^{\mathrm{Fo}} \varphi^*(\tau)\exp[s_i(\mathrm{Fo}-\tau)]\,d\tau, \tag{12}$$

where $\varphi^*(\mathrm{Fo})$ is the original of $T_0 - s\bar{\varphi}(s)$. This is a general scheme for complex use of integral transformation and the projection method in heat transfer.

To compare the approximate solutions with known exact ones, we first consider a problem with constant specific heat and thermal conductivity. For $\beta_1 = \beta_2 = 0$ the coefficients in the truncated first-order system

$$(A_{11} + B_{11}s)\,\bar{a}_1(s) = [T_0 - s\bar{\varphi}(s)]\,D_1 \tag{13}$$

are found in the form

$$A_{11} = -\int_0^1 \frac{d^2\psi_1}{d\xi^2}\psi_1 d\xi = 2\int_0^1\left[\frac{\mathrm{Bi}+2}{\mathrm{Bi}} - (1-\xi)^2\right]d\xi = \frac{4(\mathrm{Bi}+3)}{3\mathrm{Bi}},$$

$$B_{11} = \int_0^1 \psi_1^2 d\xi = \frac{4(2\mathrm{Bi}^2 + 10\mathrm{Bi} + 15)}{15\mathrm{Bi}^2},\quad D_1 = \frac{2(\mathrm{Bi}+3)}{3\mathrm{Bi}},$$

whence

$$\bar{a}_1(s) = \frac{A(\mathrm{Bi})[T_0 - s\bar{\varphi}(s)]}{2[s + A(\mathrm{Bi})]}, \tag{14}$$

where

$$A(\mathrm{Bi}) = \frac{5\mathrm{Bi}(\mathrm{Bi}+3)}{2\mathrm{Bi}^2 + 10\mathrm{Bi} + 15}. \tag{15}$$

We put $\varphi(\mathrm{Fo}) = T_c > T_0\left(\bar{\varphi}(s) = \frac{T_c}{s}\right)$ and then the temperature distribution within the soil for a constant air temperature is found from

$$T(\xi,\ \mathrm{Fo}) = T_c + \frac{(T_0 - T_c)A(\mathrm{Bi})}{2}\exp[-A(\mathrm{Bi})\,\mathrm{Fo}]\left[\frac{\mathrm{Bi}+2}{\mathrm{Bi}} - (1-\xi)^2\right]. \tag{16}$$

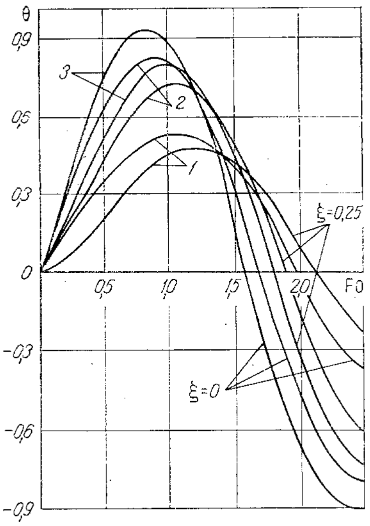

Fig. 1

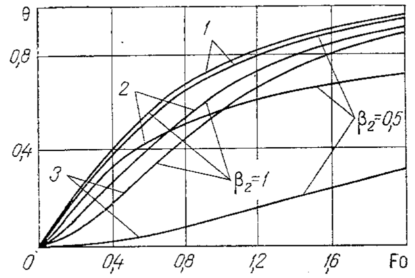

Fig. 2

Fig. 1. Variation in the relative excess temperature at the surface ($\xi=0$) and within the soil ($\xi=0.25$) for harmonic variation in the air temperature: 1) Bi = 1; 2) 4; 3) 10.

Fig. 2. Variation in temperature in the soil for an exponential rise in air temperature in the greenhouse: 1) $\xi=0$; 2) 0.25; 3) 0.5.

The relative excess temperature provided by the solution of (16) is written as

$$\theta(\xi,\ \mathrm{Fo})=\frac{T(\xi,\ \mathrm{Fo})-T_0}{T_c-T_0}=1-\frac{A(\mathrm{Bi})}{2}\left[\frac{\mathrm{Bi}+2}{\mathrm{Bi}}-(1-\xi)^2\right]\exp\left[-A(\mathrm{Bi})\,\mathrm{Fo}\right]. \tag{17}$$

Numerical analysis shows that the approximate solution of (17) is virtually the same as the exact one for the range $0<\mathrm{Bi}\leq 1$; the deviation from the exact solution increases monotonically with Bi and attains its maximum value for $\mathrm{Bi}=\infty$ (boundary conditions of the first kind). Examination of this limiting case, where the temperature at the surface $x=0$ is constant at T_c, gives good agreement with the exact solution for $\mathrm{Fo}\geq 0.05$.

Figure 1 shows variation in A(Bi), which governs the rate of exponential stabilization of the temperature pattern, and provides comparison with the square of the first root μ_1^2 of the characteristic equation $\mathrm{ctg}\,\mu=\mu/\mathrm{Bi}$; as Bi varies from 0 to 1, the value of A(Bi) exceeds μ_1^2, and the error increases monotonically from zero to 0.08% at the point Bi = 1. For Bi = 10, $\mathrm{Bi}=\infty$ the corresponding errors in A(Bi) are 1 and 1.3%. Therefore, the temperature stabilization in the approximate solutions gives good agreement with the stabilization in the exact solution.

Calculation of the temperature for the stepwise change in temperature in the boundary conditions at the initial instant, as in the solution of (17), produces the largest errors; if there is continuous variation in $\varphi(t)$ with the initial distribution $T_0(\lim\limits_{t\to 0}\varphi(t)=T_0)$, then the error in calculating the temperature becomes less.

We now determine the temperature distribution in the soil for harmonic variation in the air temperature in the greenhouse:

$$\varphi(t)=\mathrm{T}_0+\Delta\mathrm{T}\sin 2\pi\nu t=T_0+\Delta T\sin\omega\,\mathrm{Fo},\quad \omega=\mathrm{Pd}=\frac{2\pi\nu l^2}{a}.$$

We substitute the value

$$\bar{\varphi}(s)=\frac{T_0}{s}+\frac{\Delta T\omega}{s^2+\omega^2}$$

into (14) to get

$$\bar{a}_1(s)=-\frac{A(\mathrm{Bi})}{2}\,\frac{\Delta\mathrm{T}\omega s}{(s+A)(s^2+\omega^2)}=\frac{A(\mathrm{Bi})\,\Delta\mathrm{T}}{2}\left[\frac{B}{s+A}+\frac{Ds+E}{s^2+\omega^2}\right].$$

After we have determined B, D, and E by means of undefined coefficients we get

$$\bar{a}_1(s)=\frac{A(\mathrm{Bi})\,\omega\Delta T}{2}\left\{\frac{A(\mathrm{Bi})}{[A^2(\mathrm{Bi})+\omega^2](s+A)}-\frac{1}{A^2+\omega^2}\left[\frac{A(\mathrm{Bi})\,s}{s^2+\omega^2}+\frac{\omega^2}{s^2+\omega^2}\right]\right\},$$

whence

$$a_1(\mathrm{Fo})=\frac{A(\mathrm{Bi})\,\omega\Delta T}{2}\left\{\frac{A(\mathrm{Bi})}{A^2+\omega^2}\exp\left[-A(\mathrm{Bi})\,\mathrm{Fo}\right]-\frac{1}{A^2+\omega^2}\left[A\cos\omega\,\mathrm{Fo}+\omega\sin\omega\,\mathrm{Fo}\right]\right\}.$$

We put

$$\frac{A}{\sqrt{A^2+\omega^2}}=\sin\varphi_0,\quad \frac{\omega}{\sqrt{A^2+\omega^2}}=\cos\varphi_0,$$

and then the relative excess temperature is found from

$$\theta(\xi,\ \mathrm{Fo},\ \mathrm{Bi})=\frac{T(\xi,\ \mathrm{Fo})-T_0}{\Delta T}=\sin\omega\,\mathrm{Fo}+\frac{A(\mathrm{Bi})}{2}\left\{\frac{\sin 2\varphi_0}{2}\times\right.$$

$$\left.\times\exp(-A(\mathrm{Bi})\,\mathrm{Fo})-\cos\varphi_0\sin(\omega\,\mathrm{Fo}+\varphi_0)\right\}\left[\frac{\mathrm{Bi}+2}{\mathrm{Bi}}-(1-\xi)^2\right]. \tag{18}$$

Figure 1 shows the variation in θ for Bi = 1, 4, 10 at the points $\xi = 0,\ 0.25$; if the values of the thermal diffusivity, Bi, and thickness l are known for a particular soil one can calculate the temperature from (18) in terms of the dimensional coordinates x and time t.

The availability of (14) enables one to find the solution for any other laws of variation in the air temperature.

We now consider the problem of (2) and (3) with $c(x,\ m_1)=c_0(1+m_1x)$, $\lambda(x,\ m_2)=\lambda_0(1+m_2x)$, the values being taken from [2].

In our symbols we have

$$1+f_c(\xi,\ \beta_1)=1+\beta_1\xi,\quad 1+f_\lambda(\xi,\ \beta_2)=1+\beta_2\xi.$$

The coefficients for the first-order system of (9) are

$$A_{11}=\frac{4(\mathrm{Bi}+3)+\beta_2\mathrm{Bi}}{3\mathrm{Bi}},\quad B_{11}=\frac{8(2\mathrm{Bi}^2+10\mathrm{Bi}+15)+\beta_1(11\mathrm{Bi}^2+50\mathrm{Bi}+60)}{30\mathrm{Bi}^2},$$

$$D_1=\frac{8(\mathrm{Bi}+5)+\beta_1(5\mathrm{Bi}+12)}{12\mathrm{Bi}},$$

and the solution is

$$\bar{a}_1(s)=\frac{[T_0-s\bar{\varphi}(s)]\,D(\mathrm{Bi},\ \beta_1)}{s+A(\mathrm{Bi},\ \beta_1,\ \beta_2)}, \tag{19}$$

where

$$A(\mathrm{Bi},\ \beta_1,\ \beta_2)=\frac{10\mathrm{Bi}\,[4(\mathrm{Bi}+3)+\beta_2\mathrm{Bi}]}{8(2\mathrm{Bi}^2+10\mathrm{Bi}+15)+\beta_1(11\mathrm{Bi}^2+50\mathrm{Bi}+60)}; \tag{20}$$

$$D(\mathrm{Bi},\ \beta_1)=\frac{5\mathrm{Bi}\,[8(\mathrm{Bi}+3)+\beta_1(5\mathrm{Bi}+12)]}{2[8(2\mathrm{Bi}^2+10\mathrm{Bi}+15)+\beta_1(11\mathrm{Bi}^2+50\mathrm{Bi}+60)]}. \tag{21}$$

If the air temperature in the greenhouse rises exponentially in the form $\varphi(\mathrm{Fo})=T_0+(T_c-T_0)[1-\exp(-\mathrm{Pd}\,\mathrm{Fo})]$, we substitute the value $\bar{\varphi}(s)=\frac{T_0}{s}+\frac{\mathrm{Pd}(T_c-T_0)}{s(s+\mathrm{Pd})}$ into (19) to get

$$\bar{a}_1(s)=\frac{D(\mathrm{Bi},\ \beta_1)(T_c-T_0)\,\mathrm{Pd}}{A(\mathrm{Bi},\ \beta_1,\ \beta_2)-\mathrm{Pd}}\left[\frac{1}{s+A(\mathrm{Bi},\ \beta_1,\ \beta_2)}-\frac{1}{s+\mathrm{Pd}}\right].$$

Transfer to the originals gives

$$\theta(\xi,\ \mathrm{Fo},\ \mathrm{Bi},\ \beta_1,\ \beta_2)=\frac{T(\xi,\ \mathrm{Fo})-T_0}{T_c-T_0}=1-\exp(-\mathrm{Pd}\,\mathrm{Fo})+$$

$$+\frac{D(\mathrm{Bi},\ \beta_1)\,\mathrm{Pd}}{A(\mathrm{Bi},\ \beta_1,\ \beta_2)-\mathrm{Pd}}\{\exp[-A(\mathrm{Bi},\ \beta_1,\ \beta_2)\,\mathrm{Fo}]-\exp(-\mathrm{Pd\,Fo})\}\left[\frac{\mathrm{Bi}+2}{\mathrm{Bi}}-(1-\xi)^2\right]. \quad (22)$$

Figure 2 shows the variation in θ for Bi = 4, Pd = 2, and a constant specific heat, $\beta_1 = 0$, $\beta_2 = 0.5$, 1 at the points $\xi = 0$, 0.25, 0.5.

With boundary conditions of the first kind (Bi = ∞) we get from the solution to (22) for the case $\beta_1 = 0$, $\beta_2 \neq 0$ that

$$\theta(\xi,\ \mathrm{Fo},\ \beta_2) = 1-\exp(-\mathrm{PdFo})+\frac{5\mathrm{Pd}}{4\,[2.5+0.625\beta_2-\mathrm{Pd}]}\{\exp[-(2.5+0.625\beta_2)\,\mathrm{Fo}]-\exp(-\mathrm{PdFo})\}(2\xi-\xi^2). \quad (23)$$

If there is a decaying amplitude for the harmonic oscillation of the air temperature $\varphi(\mathrm{Fo}) = T_0 + \Delta T\exp(-\mathrm{PdFo})\cdot\sin\omega\,\mathrm{Fo}$, then the relative excess temperature in the soil is given by

$$\theta(\xi,\ \mathrm{Fo},\ \mathrm{Bi},\ \beta_1,\ \beta_2) = \frac{T(\xi,\ \mathrm{Fo})-T_0}{\Delta T} = \exp(-\mathrm{Pd\,Fo})\sin\omega\,\mathrm{Fo} +$$

$$+\frac{D}{\omega^2+(\mathrm{Pd}-A)^2}\{A\omega\exp(-A\,\mathrm{Fo})-\exp(-\mathrm{Pd\,Fo})[A\omega\cos\omega\,\mathrm{Fo}+(\mathrm{Pd}(\mathrm{Pd}-A)+\omega^2)\sin\omega\,\mathrm{Fo}]\}\left[\frac{\mathrm{Bi}+2}{\mathrm{Bi}}-(1-\xi)^2\right] \quad (24)$$

In the solutions to (22)-(24), the coefficient for the rate of exponential stabilization of the temperature pattern defined by (20) increases with β_2, the correcting parameter for the nonuniformity in the thermal conductivity, and it is inversely proportional to β_1, which is in accordance with the physics of thermal conduction. In conclusion we note that these studies show that it is possible to solve the heat-conduction problem with any other empirical formulas for the variable specific heat and thermal conductivity, which are dependent on the coordinates, and these constitute models for heat-transfer processes in the soil.

NOTATION

Fo = at/l^2, dimensionless time (Fourier number); s, integral Laplace transform parameter; Pd = $2\pi\nu l^2/a$, Predvoditelev number; $\xi = x/l$, dimensionless coordinate; Bi, Boit number.

LITERATURE CITED

1. D. A. Kurtener and A. F. Chudnovskii, Calculation and Control of Thermal Conditions in Open and Covered Soil [in Russian], Gidrometeoizdat, Leningrad (1969), p. 296.
2. A. F. Chudnovskii, Thermophysics of Soils [in Russian], Nauka, Moscow (1976), p. 352.
3. P. V. Tsoi, Methods of Calculating Individual Problems in Heat and Mass Transfer [in Russian], Énergiya, Moscow (1971), p. 375.
4. A. V. Lykov, Theory of Thermal Conduction [in Russian], Vysshaya Shkola, Moscow (1967), p. 592.
5. R. Bairamov, N. Gurbanov, and L. E. Rybakova, "A simplified method of thermal calculation for solar greenhouses on the basis of nonstationary operation," Geliotekhnika, No. 3, 45-49 (1973).